An Overview on Behavioural Theory to Systems and Control

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Presentation: At A glance

- Objectives
- A look behind to control theory
- Behavioural Approach
- People in behavioural theory
- Recent Achievements
- Concluding Remarks

❑ Objectives

- Control theory is an interdisciplinary branch of engineering and mathematics.
- It deals with influencing the behavior of dynamical systems.
- Behavioural theory is a newly introduced area in systems and control.
- Our aim is to introduce the very basic notions of this theory
- Its scopes in the field of systems and control.
- We also investigate the recent works in this area.

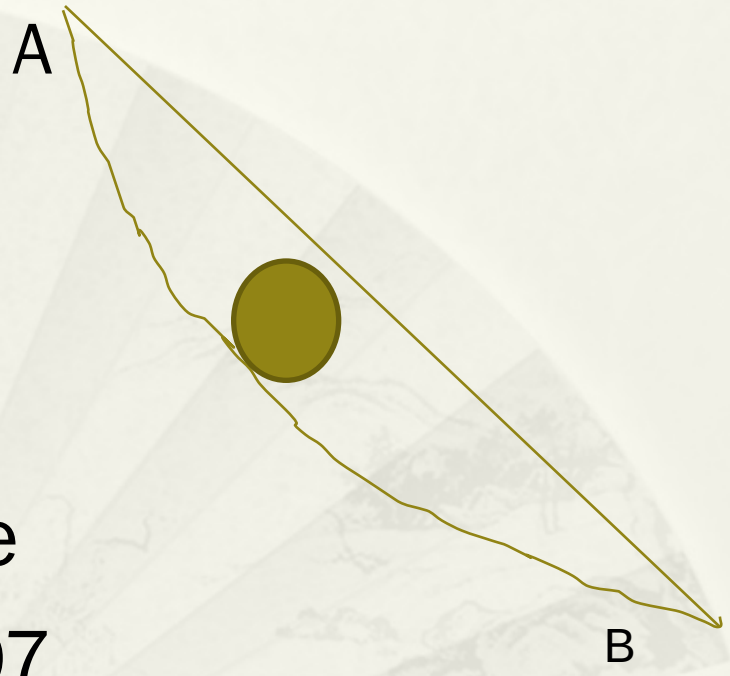
❑ A look behind to control theory

- The concept of control theory came to light more than three century ago after the publication of Johann Bernouli's solution of the brachystochrone problem in 1697.

- The problem was solved by Newton and Bernouli independently, Bernouli was the first who articulated the principle of optimality .

- Later on, various optimality principles were formulated by Pierre de Fermat (1601-1665) (in optics), Carl Friedrich Gauss (1777-1855), Jean d'Alembert (1717-1783), Euler, Lagrange and Hamilton, and Albert Einstein (1879-1955) (in mechanics).

❑ A look behind to control theory (Contd.)



Johann Bernouli's solution of the brachystochrone problem in 1697.

❑ A look behind to control theory (Contd.)

- In 1957 Richard Bellman formulated the *dynamic programming* principle to the optimal control of discrete-time systems.

- In 1958 Lev Pontryagin developed the *maximum principle* for solving nonlinear optimal control problems.

❑ Behavioural Approach

- The field of systems and control has come a long way in the last 50-60 years.

- The addition of behavioural theory in systers and control is a new era.

- In the present century, the pioneer in the *behavioural systems theory* is J. C Willems. He has introduced the concept of *behavioural systems theory*).

❑ Behavioural Approach (Contd.)

- In the 1960s, control was considered an electrical engineering subject, even though many applications of control involved mechanical machines or chemical processes.
- This required a lot of mathematical methods to fomulating the models.
- Behavioural theory is completely developed based on the different mathematical models of the dynamical systems.

❑ Behavioural Approach (Contd.)

❖ Mathematical Model:

- A ***mathematical model*** is a pair $(\mathbb{U}, \mathbb{B})$

with $\mathbb{U}$ the ***universum its*** elements are called *outcomes* and $\mathfrak{B}$ the *behaviour* of the model, a subset of the universum.

- Example: Let $\mathbb{U}$ = {ice, water, steam} × [-273, ∞)

Then the model is

$\mathfrak{B}$ = ({ice} × [-273,0]) U ({water} × [0,100]) U ({steam}× [100,∞)).

❑ Behavioural Approach (Contd.)

❖ Behavioural equation

- Let $\mathbb{U}$ be a universum, $\mathbb{E}$ an abstract set, called the ***equating spaces,*** and let $f_1,\ f_2 : \mathbb{U} \longrightarrow \mathbb{E}$

Then the mathematical model $(\mathbb{U}, \mathbb{B})$
with $\mathfrak{B} = \{u \in \mathbb{U} : f_1(u) = f_2(u)\}$
is said to be described by **behavioural equation.**

- The equality indicates the equilibrium state.

❑ Behavioural Approach (Contd.)

- Behavioural equation

- Around 1960, the basic model for studying dynamics in control was shifted from the differential equation

$$p\left(\frac{d}{dt}\right) y = q\left(\frac{d}{dt}\right) u$$

with p and q real polynomials, to a transfer function

$$\dot{x} = f(x, u, t), \; y = h(x, u, t)$$

❑ Behavioural Approach (Contd.)

- Differential equation

- In the late 1970s, Jan C. Willems gave a detailed treatment of the highly structured linear time-invariant systems starting with the equations

$$\dot{x} = Ax + Bu,\ y = Cx + Du$$

Or even

$$\dot{x} = f(x,\ u),\ y = h(x,\ u)$$

❑ People in behavioural theory

- Jan C. Willems

(http://homes.esat.kuleuven.be/~jwillems)

- Pioneer of archetype of behavioural theory
- Books : 3
- No. of Publications: more than 120 (till 2008)
- Selected Conference Articles: 11
- Articles that only appeared in Conference Proceedings, Summer School Lecture Notes, etc.: 181
- Edited Books: 7
- Chapters in Books and Festschrifts : 29

❑ People in behavioural theory (Contd.)

- P. M. Rocha

(http://paginas.fe.up.pt/~mprocha/)

- One of the significantly contributed author in this area since 1990.
- A lot of contributions in one-dimensional and multidimensional behavioral systems and control for biomedical applications.
- No. of chapters in books: 12
- No. of Journal papers: 27
- No. of papers in conference proceedings: 52

❑ People in behavioural theory (Contd.)

- Paolo Rapisarda

- Control and System Theory, Identification
- BDFs and QDFs with two-variable polynomials

- H. L. Trentelman

- Robust control of linear systems, and the modeling
- Representation, and control of systems in a behavioral framework.

- Madhu N. Belur

- Established the connection between freedom of disturbances in the controlled system, and regularity of interconnections.

❑ Recent Achievements

- One-dimensional and multidimensional behavioral systems
- Algorithmic aspects of the identification of linear systems
- Model reduction and approximation for linear systems
- Simulation of linear dynamical systems
- Interpolation and its applications to problems in systems and control theory
- Algorithmic aspects of polynomial and of behavioral control
- Theory and application of quadratic differential forms.

❑ Concluding Remarks

- The behavioral approach to dynamical systems, introduced only 20 years ago by J.C. Willems, is nowadays a well established branch of Systems and Control.
- For the increasing need of applications, it is perceived as well-understood and well-explored by the specialists of this area.
- Playing a dominating role in control theory problems such as the modelling of linear Hamiltonian systems, and an equipartition of energy principle.
- We hope this field will open a new horizon in systems and control in the next few decades.

Thanks for your kind attention

QUESTIONS PLEASE ?